A PROJECT WORK ON

'NUMERICAL ANALYSIS'

SUBMITTED IN PARTIAL FULFILLMENT FOR AWARD OF DEGREE OF

BACHELOR OF SCIENCE IN MATHEMATICS

UNDER THE GUIDANCE OF

B.REVATHI

LECTURER IN MATHEMATICS

BY

III B.Sc M.P.C S

DEPARTMENT OF MATHEMATICS

S.CH.V.P.M.R GOVT DEGREE COLLEGE

GANAPAVARAM

ACADEMIC YEAR 2021-22

DEPARTMENT OF MATHEMATICS

S.CH.V.P.M.R GOVT DEGREE COLLEGE

GANAPAVARAM

CERTIFICATE

This is to certify that the work, incorporated in this project titled "NUMERICAL ANALYSIS" submitted by III B.Sc MPCS have been carried out under my supervision during the academic year 2021-22. K SRAVANI

Place : Ganapavaram

Date : 02/08/22

Lecturer in MATHEMATICS

DEPARTMENT OF MATHEMATICS

S.CH.V.P.M.R GOVT DEGREE COLLEGE

GANAPAVARAM

DECLARATION

I hereby declare that the project titled "NUMERICAL ANALYSIS" work was done by me under the guidance of B.REVATHI, Lecturer in Mathematics and submitted to S.CH.V.P.M.R Govt degree college, Ganapavaram for the award of B.Sc degree from Adikavi Nannaya University, Rajamahendravaram.

Place : Ganapavaram

Date : 6|8|2022

K. Sravani
Signature of the Student

BATCH -III

S.No	Regd No
1	193337102040
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ABSTRACT

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The main aim of this project is to produce the applications of finite differences on real life which we studied in sixth semester.

This project is divided into three chapters.

Chapter 1 is introduction, in which we discuss the existing literature that is needed to develop the project.

In chapter 2 we discuss the working rule and workout examples which support to the chapter 3

In chapter 3, we produce the numerical analysis and its applications on real time by taking examples of stastical data. now we explain Newton's forward and backward interpolation formula by taking an example of finding population of a particular year. Further now we explain the Gauss backward interpolation fomula by taking an example to find sales in a particular year and then we explained Stirlings interpolation formula by taking an example to find the poverty number (%) in a particular year.

CHAPTER – 1
INTRODUCTION

Chapter-1

INTRODUCTION

INTERPOLATION:

Let $y=f(x)$ be a function of x to compute the value of y corresponding to value of x where x lies between given data is called interpolation

FINITE DIFFERENCE:

Let $y=f(x)$ is a function of single variable in x and $y_0, y_1, y_2 \ldots y_n$ are the values of y corresponding to the values $x_0, x_1, x_2 \ldots x_n$ of x respectively.

Therefore, the final differences are three types

- Forward differences (Δ)
- Backward differences (∇)
- Central difference (δ)

Forward differences (Δ):

let $y=f(x)$ is a function of x and $y_0, y_1, y_2 \ldots y_n$ are the values of y corresponding to the values x_0, $x_1 = x_0 + h$, $x_2 = x_0 + 2h \ldots x_n = x_0 + nh$ of x respectively . The difference $y_1 - y_0$, $y_2 - y_1 \ldots y_n - y_{n-1}$ are called first ordered

Forward difference of y given as

$\Delta y_0 = y_1 - y_0, \Delta y_1 = y_2 - y_1 \ldots\ldots\ldots \Delta y_{n-1} = y_n - y_{n-1}.$

Therefore $\Delta y_r = y_{r+1} - y_r \quad r=0, 1, 2\ldots.$

Similarly

$\Delta^2 y_r = \Delta y_{r+1} - \Delta y_r \quad r=0,1,2\ldots..$

And

$\Delta^n y_r = \Delta^{n-1} y_{r+1} - \Delta^{n-1} y_r \quad \text{if } r=0, 1, 2\ldots..$

Forward difference table:

x	y=f(x)	Δ y	Δ^2 y	Δ^3 y	Δ^4 y
X_0	y_0				
		$y_1-y_0=\Delta y_0$			
x_1	y_1		$\Delta y_1-\Delta y_0=\Delta^2 y_0$		
		$y_2-y_1=\Delta y_1$		$\Delta^2 y_1-\Delta y_0=\Delta^3 y_0$	
x_2	y_2		$\Delta y_2-\Delta y_1=\Delta^2 y_1$		$\Delta^3 y_1-\Delta^3 y_0=\Delta^4 y_0$
		$y_3-y_2=\Delta y_2$		$\Delta^2 y_2-\Delta y_1=\Delta^3 y_1$	
x_3	y_3		$\Delta y_3-\Delta y_2=\Delta^2 y_2$		
		$y_4-y_3=\Delta y_3$			
x_4	y_4				
.	.				
.	.				
.	.				

Backward differences (∇):let y=f(x) is a function of x and $y_0, y_1, y_2 \ldots y_n$ are the values of y corresponding to the values x_0, $x_1 = x_0 + h$, $x_2 = x_0 + 2h \ldots$ $x_n = x_0 + nh$ of x respectively . The difference y_1-y_0, $y_2-y_1 \ldots$ y_n-y_{n-1} are called first ordered backward difference of y given as

$\nabla y_1 = y_1-y_0, \nabla y_2 = y_2-y_1 \ldots\ldots\ldots \nabla y_n = y_n-y_{n-1}.$

Therefore $\nabla y_r = y_r - y_{r-1}$ r=1,2….n

Similarly

$\nabla^2 y_r = \nabla y_r - \nabla y_{r-1}$ r=2,3,…..n

And

$\nabla^n y_r = \nabla^{n-1} y_r - \nabla^{n-1} y_{r-1}$ r=n,n+1…..

Backward difference table:

x	y=f(x)	∇ y	∇^2 y	∇^3 y	∇^4 y
x_0	y_0				
		$y_1 - y_0 = \nabla y_0$			
x_1	y_1		$\nabla y_2 - \nabla y_1 = \nabla^2 y_1$		
		$y_2 - y_1 = \nabla y_1$		$\nabla^2 y_3 - \nabla^2 y_2 = \nabla^3 y_3$	
x_2	y_2		$\nabla y_3 - \nabla y_2 = \nabla^2 y_3$		$\nabla^3 y_4 - \nabla^3 y_3 = \nabla^4 y_4$
		$y_3 - y_2 = \nabla y_2$		$\nabla^2 y_4 - \nabla^2 y_3 = \nabla^3 y_4$	
x_3	y_3		$\nabla y_4 - \nabla y_3 = \nabla^2 y_4$		
		$y_4 - y_3 = \nabla y_3$			
x_4	y_4				
.	.				
.	.				
.	.				

Central difference(δ):

let y=f(x) is a function of x and y_0, y_1, y_2... y_n are the values of y corresponding to the values x_0, $x_1 = x_0 + h$, $x_2 = x_0 + 2h$... $x_n = x_0 + nh$ of x respectively. The difference y_1-y_0, y_2-y_1... y_n-y_{n-1} are called first ordered

central difference of y i.e.,

$\delta y_{1/2}=y_1-y_0, \delta y_{3/2}=y_2-y_1 \ldots\ldots\ldots \delta_{n-1/2}=y_n-y_{n-1}$

therefore $\delta_{r-1/2}=y_r-y_{r-1}$, r=1,2,3,.........

The differences $\delta y_{3/2}-\delta y_{1/2}, \delta y_{5/2}-\delta y_{3/2}$......... are called the second ordered center difference and we denote them as $\delta^2 y_1, \delta^2 y_2$......

therefore $\delta^2 y_r = \delta y_{r+1/2}-\delta y_{r-1/2}$ r=1,2........

similarly$\delta^n y_r=\delta^{n-1} y_{r+1/2}-\delta^{n-1} y_{r-1/2}$ if is even

$\delta^n y_{r-1/2}=\delta^{n-1} y_r-\delta^{n-1} y_{r-1}$ if n is odd

Center difference table

x	y=f(x)	δ y	δ^2 y	δ^3 y	δ^4 y
x_0	y_0				
		$y_1-y_0=\delta y_{1/2}$			
x_1	y_1		$\delta y_{3/2}-\delta y_{1/2}=\delta^2 y_1$		
		$y_2-y_1=\delta y_{3/2}$		$\delta^2 y_2-\delta^2 y_1=\delta^3 y_{3/2}$	
x_2	y_2		$\delta y_{5/2}-\delta y_{3/2}=\delta^2 y_2$		$\delta^3 y_{5/2}-\delta^3 y_{3/2}=\delta^4 y_2$
		$y_3-y_2=\delta y_{5/2}$		$\delta^2 y_3-\delta^2 y_2=\delta^3 y_{5/2}$	
x_3	y_3		$\Delta y_{7/2}-\delta y_{5/2}=\delta^2 y_3$		
		$y_4-y_3=\delta y_{7/2}$			
x_4	y_4				
.	.				
.	.				
.	.				

Symbolic relations and separation of symbols:

1. Forward difference operator(Δ):

The forward difference operator Δ is defined by the equation

$$\Delta f(x)=f(x+h)-f(x)$$

2. Backward difference operator (∇):

The difference operator backward∇ is defined by the equation $\nabla f(x)=f(x)-f(x-h)$

3. Center difference operator (δ):

The difference operator is defined by the equation $\delta f(x)=f(x+h/2)-f(x-h/2)$

4. Average operator (μ):

The average operator μ is defined by the equation $\mu(f.x)=1/2[f(x+h/2)+f(x-h/2)]$

5. Shift operator (E):

The shift operator E is defined by the equation

$E(f(x)) = f(x+h)$ or $E(y_r) = y_{r+1}$

The 2nd ordered shift operator

$$\begin{aligned} E^2(f(x)) &= E(E\,f(x)) \\ &= E(f(x+h)) \\ &= f(x+h+h) \\ &= f(x+2h) \end{aligned} \quad \text{or} \quad \begin{aligned} E^2 y_0 &= E(E(y_r)) \\ &= E(y_{r+1}) \\ &= y_{r+1+1} \\ &= y_{r+2} \end{aligned}$$

$E^n(f(x)) = f(x+nh)$ or $E^n y_r = y_{r+n}$

6. Inverse Shift operator (E^{-1}):

The Inverse shift operator E^{-1} is defined by the equation

$E^{-1}(f(x)) = f(x-h)$ or $E^{-1}(y_r) = y_{r-1}$

The 2nd ordered Inverse shift operator

$E^{-2}(f(x)) = E^{-1}(E f(x))$ or $E^{-2} y_0 = E^{-1}(E^{-1}(y_r))$

$= E^{-1}(f(x-h))$ $= E^{-1}(y_{r-1})$

$= f(x-h-h)$

$= y_{r-1-1}$

$= f(x-2h)$ $= y_{r-2}$

$E^{-n}(f(x)) = f(x-nh)$ or $E^{-n} y_r = y_{r+n}$

7. Identity operator (I):

The identity operator I is defined by the equation

$I(f(x))=f(x)$

8. Differential operator (D):

The Differential operator D is defined by the equation

$$Df(x) = d/dx\,(f(x))$$

Second order

$$D^2 f(x)=d^2/dx^2\,(f(x))$$

.

.

.

n^{th}order

$$D^n f(x)=d^n/dx^n(f(x))$$

9. Inverse differential operator (or) Integral operator :

The inverse differential operator or integral operator I is defined by the equation

$$J(f(x)) = D^{-1}f(x)$$

- ## Fundamental theorem infinite difference:

If $f(x)$ is a polynomial of degree n and the values of x are equally spaced or equal intervals then the n^{th} order difference of $f(x)$ is constant and the $(n+1)^{th}$ difference of $f(x)$ is 0.

Or

If $f(x) = a_0+a_1x+a_2x+....+a_nx_n= \Sigma^n_{i=0} a_ix_i$ is a polynomial of degree n. Then $\Delta^n f(x)=a_n n!h^n$ and $\Delta^{n+1}f(x)=0$.

Chapter-2

INTERPOLATION WITH EQUAL INTERVALS

Chapter-2

INTERPOLATION WITH EQUAL INTERVALS

Newton's forward interpolation formulae:

Statement: If y= f(x) is a function and y_0 = f(x) , y_1 = f(x), y_0 = f(x),y_1= f(x_1), y_2= f(x_2),….. y_n= f(x_n), are the values of y= f(x) corresponding to the values x_0, x_1=x_0+h, x_2=x_0+2h,…….xn= x_0+nh of augment x.

$$Y_u= y_0+u\Delta y_0+\frac{u(u-1)\Delta^2 y_0}{2!}+\frac{u(u-1)(u-2)\Delta^3 y_0}{3!}+\ldots\ldots+\frac{u(u-1)\ldots..(u-(n-1))\Delta^n y_0}{n!}$$

Where $u=\frac{x-x_0}{h}$

Proof: given that y= f(x) is a function and y_0 = f(x) , y_1 = f(x), y_0 = f(x),y_1= f(x_1), y_2= f(x_2),….. y_n= f(x_n), are the values of y= f(x) corresponding to the values x_0, x_1=x_0+h, x_2=x_0+2h,…….x_n= x_0+nh of augment x.

Consider n^{th}degree polynomial

$$f(x) = A_0+A_1(x-x_0)+A_2(x-x_0)(x-x_1)+\ldots+A_n(x-x_0)(x-x_1)\ldots\ldots(x-x_{n-1}) \rightarrow (1)$$

where A_0,A_1…..A_n are constants

put x=x_0 in equation 1

$$f(x_0) = A_0+0+0+\ldots\ldots\ldots.+0$$

$$A_0= f(x_0)=y_0$$

put x=x_1 in equation 1 we get

$$f(x_1) = A_0+A_1(x-x_0)+0+\ldots\ldots\ldots+0$$

$y_1 = y_0 + A_1(x_0+h-x_0)$

$y_1 - y_0 = A_1 h$

$A_1 h = \Delta y_0$

$A_1 = 1/h\ \Delta y_0$

put $x=x_2$ in equation 1 we get

$f(x_2) = A_0+A_1(x_2-x_0)+A_2(x_2-x_0)\ (x_2-x_1)+0\ldots\ldots\ldots+0$

$y_2 = y_0 + 1/h\ \Delta y_0(x_0+2h-x_0)+A_2(x_0+2h-x_0)+\ldots.+\ (x_0+2h-x_h)$

$y_2 = y_0+2(y_1-y_0)+A_2 2h^2$

$A_2 2h^2 = y_2-y_0-2y_1+2y_0$

$A_2 = 1/2h^2(y_2-2y_1+y_0)$

$A_2 = ½!h^2\Delta^2 y_0$

Similarly

$A_3 = 1/(3!h^3)\Delta^3 y_0$

and $A_4 = 1/(4!h^4)\Delta^4 y_0$

.

.

. $A_n = 1/n!h^n\Delta^n y_0$

Substitute the values of $A_0,A_1\ldots..A_n$ in equation 1 we get

$f(x) = y_0+1/h\ \Delta y_0(x-x_0)+ ½!h^2\ \Delta^2 y_0(x-x_0)\ (x-x_1)+\ldots.+ 1/n!h^n\Delta^n y_0(x-x_0)\ (x-x_1)\ldots..$

$(x-x_{n-1})$

since $u=\underline{x-x_0}$
$\quad\quad\quad\quad h$

$x=x_0+uh$

$f(x_0+uh) = y_0+1/h\,\Delta y_0(x_0+uh-x_0)+ \frac{1}{2!h^2}\Delta^2 y_0(x_0+uh-x_0)(x_0+uh-x_1)+\ldots+$

$1/n!h^n\Delta^n y_0(x_0+uh-x_0)(x_0+uh-x_1)\ldots(x_0+uh-x_{n-1})$

$$Y_u = y_0+u\Delta y_0+\frac{u(u-1)}{2!}\Delta^2 y_0+\frac{u(u-1)(u-2)}{3!}\Delta^3 y_0+\ldots\ldots+\frac{u(u-1)\ldots(u-(n-1))}{n!}\Delta^n y_0$$

$$Y_u = y_0+uc_1\Delta y_0+uc_2{}^2y_0+uc_3\Delta^3 y_0+\ldots\ldots+uc_n\Delta^n y_0$$

Newton's backward interpolation formulae:

If $y= f(x)$ is a function and $y_0 = f(x_0)$, $y_1 = f(x_1)$, $y_2 = f(x_2)$…… $y_n = f(x_n)$ are the values of $y= f(x)$ corresponding to the values x_0, $x_1=x_0+h$, $x_2=x_0+2h$,…….$xn= x_0+nh$ of augment x.

$$Y_u= y_n+u\nabla y_n+\frac{u(u+1)}{2!}\nabla^2 y_n+\frac{u(u+1)(u+2)}{3!}\nabla^3 y_n+\ldots\ldots+\frac{u(u+1)\ldots(u+(n-1))}{n!}\nabla^n y_n$$

Where $u=\frac{x-x_n}{h}$

Gauss' forward interpolation formulae:

Let $y= f(x)$ is a function and $y_{-3} = f(x_{-3})$, $y_{-2} = f(x_{-2})$, $y_{-1} = f(x_{-1})$, $y_0 = f(x_0)$, $y_1 = f(x_1)$, $y_2 = f(x_2)$, $y_3 = f(x_3)$…… are the values of $y= f(x)$ corresponding to the values x are
$x_{-3}=x_0-3h$, $x_1=x_0+h$, $x_{-2}=x_0-2h$,……. $x_{-1}=x_0-h$, x_0, $x_1= x_0+h$, $x_2= x_0+2h$, $x_3= x_0+3h$……then

$$Y_u= y_0+u\Delta y_0+\frac{u(u-1)}{2!}\Delta^2 y_{-1}+\frac{u(u+1)(u-1)}{3!}\Delta^3 y_{-1}+\frac{u(u+1)u(u-1)(u-2)}{4!}\Delta^4 y_{-2}+\ldots\ldots$$

Where $u=\frac{x-x_0}{h}$

Proof: Given that $y= f(x)$ is a function of x which takes the values…..
$y_{-3} = f(x_{-3})$, $y_{-2} = f(x_{-2})$, $y_{-1} = f(x_{-1})$, $y_0 = f(x_0)$, $y_1 = f(x_1)$, $y_2 = f(x_2)$, $y_3 = f(x_3)$……corresponding to the values of x are

$x_{-2}=x_0-2h$, $x_1=x_0+h$, $x_{-2}=x_0-2h$, $x_{-1}=x_0-h, x_0$, $x_1= x_0+h$, $x_2= x_0+2h$, $x_3= x_0+3h$……and $u=\frac{x-x_0}{h}$

By NFIF

$$Y_u= y_0+u\Delta y_0+\frac{u(u-1)\Delta^2 y_0}{2!} +\frac{u(u-1)(u-2)\ \Delta^3 y_0}{3!}+\frac{u(u-1)u(u-2)(u-3)\ \Delta^4 y_0}{4!}+\ldots\ldots\ldots$$

Difference table:

X	Y	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
.	.						
.	.						
.	.						
x_{-3}	y_{-3}						
		Δy_{-3}					
x_{-2}	y_{-2}		$\Delta^2 y_{-3}$				
		Δy_{-2}		$\Delta^3 y_{-3}$			
x_{-1}	y_{-1}		$\Delta^2 y_{-2}$		$\Delta^4 y_{-3}$		
		Δy_{-1}		$\Delta^3 y_{-2}$		$\Delta^5 y_{-3}$	
x_0	y_0 ↘		↗ $\Delta^2 y_{-1}$		↗ $\Delta^4 y_{-2}$		↗ $\Delta^6 y_{-3}$
		Δy_0	↘	$\Delta^3 y_{-1}$	↘	$\Delta^5 y_{-2}$	
x_1	y_1		$\Delta^2 y_0$		$\Delta^4 y_{-1}$		
		Δy_1		$\Delta^3 y_0$			
x_2	y_2		$\Delta^2 y_1$				
		Δy_2					
x_3	y_3						
.	.						
.	.						
.	.						

From table

$X_0 \ldots\ldots y_0 \ldots\ldots\ldots\ldots \Delta^2 y_{-1} \ldots\ldots\ldots\ldots \Delta^4 y_{-2} \ldots\ldots\ldots \Delta^6 y_{-3}$

$\Delta y_0 \qquad \Delta^3 y_{-1} \qquad \Delta^5 y_{-2}$

And $\Delta^2 y_0 - \Delta^2 y_{-1} = \Delta^3 y_{-1} = \Delta^2 y_0 = \Delta^2 y_{-1} - + \Delta^3 y_{-1}$

$\Delta^3 y_0 - \Delta^3 y_{-1} = \Delta^4 y_{-1} = \Delta^3 y_0 = \Delta^3 y_{-1} - + \Delta^4 y_{-1}$

Similarly

$\Delta^4 y_0 = \Delta^4 y + \Delta^6 y_{-1}$

.

.

And $\Delta^4 y_{-1} - \Delta^4 y_{-2} = \Delta^5 y_{-2} = \Delta^4 y_{-2} + \Delta^5 y_{-2} \Delta^5 y_{-1} = \Delta^5 y_{-2} + \Delta^6 y_{-2}$

From equation 1 we get

$$Y_u = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}[\Delta^2 y_{-1} + \Delta^2 y_{-1}] + \frac{u(u-1)(u-2)}{3!}[\Delta^3 y_{-1} + \Delta^4 y_{-1}] + \frac{u(u-1)(u-2)(u-3)}{4!}[\Delta^4 y_{-1} + \Delta^5 y_{-1}] + \ldots\ldots\ldots$$

$$Y_u = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_{-1} + \left[\frac{u(u-1)}{2!} + \frac{u(u-1)(u-2)}{3!}\right]\Delta^3 y_{-1} + \left[\frac{u(u-1)(u-2)}{3!} + \frac{u(u-1)(u-2)(u-3)}{4!}\right][\Delta^4 y_{-1}] + \ldots\ldots\ldots$$

$$Y_u = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_{-1} \frac{u(u+1)+(u-1)}{3!}\Delta^3 y_{-1} + \frac{u(u+1)(u-1)(u-2)}{4!}\Delta^4 y_{-2} + \ldots\ldots\ldots$$

This is called Gauss' forward interpolation formulae.

Gauss' backward interpolation formulae:

Let $y= f(x)$ is a function and $y_{-3} = f(x_{-3})$, $y_{-2} = f(x_{-2})$, $y_{-1} = f(x_{-1})$, $y_0 = f(x_0)$, $y_1 = f(x_1)$, $y_2 = f(x_2)$, $y_3 = f(x_3)$...... are the values of $y= f(x)$ corresponding to the values x are
$x_{-3}=x_0-3h$, $x_1=x_0+h$, $x_{-2}=x_0-2h$,....... $x_{-1}=x_0-h$, x_0, $x_1= x_0+h$, $x_2= x_0+2h$, $x_3= x_0+3h$......then

$$Y_u= y_0+u\Delta y_{-1}+\frac{u(u+1)\Delta^2 y_{-1}}{2!}+\frac{u(u+1)(u-1)\,\Delta^3 y_{-1}}{3!}+\frac{u(u+1)u(u-1)(u+2)\,\Delta^4 y_{-2}}{4!}+\dots\dots$$

Where $u=\frac{x-x_0}{h}$

Striling's formula:

Let $y= f(x)$ is a function and $y_{-3}, y_{-2}, y_{-1}, y_0, y_1, y_2, \dots\dots$ are the values of x corresponding to the values x are
$x_{-3}, x_{-2}, x_{-1} x_0, x_1, x_2, x_3$......then

$$Y_u= y_0+u\left[\frac{\Delta y_0+\Delta y_{-1}}{2}\right]+\frac{u^2\Delta^2 y_{-1}}{2!}+\frac{(u^2-1)u[\Delta^3 y_{-1}+\Delta^3 y_{-2}]}{3!\quad 24!}+\frac{(u^2-1)\,u^2\Delta^4 y_{-2}}{}+\dots\dots$$

Where $u=\frac{x-x_0}{h}$

Proof: Given that $y= f(x)$ is a function and $y_{-3}, y_{-2}, y_{-1}, y_0, y_1, y_2, \dots\dots$ are the values of x corresponding to the values x are
$x_{-3}, x_{-2}, x_{-1} x_0, x_1, x_2, x_3$...... When $u=\frac{x-x_0}{h}$

x	Y	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
.	.						
.	.						
.	.						
x_{-3}	y_{-3}						
		Δy_{-3}					
x_{-2}	y_{-2}		$\Delta^2 y_{-3}$				
		Δy_{-2}		$\Delta^3 y_{-3}$			
x_{-1}	y_{-1}		$\Delta^2 y_{-2}$		$\Delta^4 y_{-3}$		
		Δy_{-1}		$\Delta^3 y_{-2}$		$\Delta^5 y_{-3}$	
x_0	y_0		$\Delta^2 y_{-1}$		$\Delta^4 y_{-2}$		$\Delta^6 y_{-3}$
		Δy_0		$\Delta^3 y_{-1}$		$\Delta^5 y_{-2}$	
x_1	y_1		$\Delta^2 y_0$		$\Delta^4 y_{-1}$		
		Δy_1		$\Delta^3 y_0$			
x_2	y_2		$\Delta^2 y_1$				
		Δy_2					
x_3	y_3						
.	.						
.	.						
.	.						

By GFIF

$$Y_u = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_{-1} + \frac{u(u+1)(u-1)}{3!}\Delta^3 y_{-1} + \frac{u(u+1)(u-1)(u-2)}{4!}\Delta^4 y_{-2} + \ldots\ldots\ldots (1)$$

By GBIF

$$Y_u = y_0 + u\Delta y_{-1} + \frac{u(u+1)}{2!}\Delta^2 y_{-1} + \frac{u(u+1)(u-1)}{3!}\Delta^3 y_{-1} + \frac{u(u+1)u(u-1)(u+2)}{4!}\Delta^4 y_{-2} + \ldots\ldots\ldots$$

By adding ((1)+(2))/(2)

$$Y_u = y_0 + u\left[\frac{\Delta y_0 + \Delta y_{-1}}{2}\right] + \frac{u(u-1+u+1)}{2!}\,\frac{\Delta^2 y_{-1}}{2} + \frac{(u^2-1)u}{3!}\left[\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right] + \frac{(u^2-1)\,u^2}{4!}\Delta^4 y_{-2} + \ldots$$

$$Y_u = y_0 + u\left[\frac{\Delta y_0 + \Delta y_{-1}}{2}\right] + \frac{u^2}{2!}\Delta^2 y_{-1} + \frac{(u^2-1)u}{3!}\left[\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right] + \frac{(u^2-1)\,u^2}{4!}\Delta^4 y_{-2} + \ldots$$

This is called Gauss' central difference formula or striling formula

Chapter-3
APPLICATIONS OF FINITE DIFFERENCES

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APPLICATIONS OF FINITE DIFFERENCES

1. The population of a town in the given below estimate the population for the years 1895 and 1925.

Year(x):	1891	1901	1911	1921	1931
Population(Thousands):	46	66	81	93	101

Sol: Let y=f(x) be the function of x

Difference table:

x	y	Δ	Δ^2	Δ^3	Δ^4
1891	46				
		20			
1901	66		-5		
		15		2	
1911	81		-3		-3
		12		-1	
1921	93		-4		
		8			
1931	101				

From the above table,

Here $x_0=1891, y_0=46, \Delta y_0=20, \Delta^2 y_0= -5, \Delta^3 y_0=2, \Delta^4 y_0= -3$

To find the population of the year 1895:

$$u = \frac{x - x_0}{h}$$

$$= \frac{1895-1891}{10}$$

$=0.4$

By NFIF

$$Y_u = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \ldots\ldots + \frac{u(u-1)\ldots..(u-(n-1))}{n!}\Delta^n y_0$$

$$f(x) = 46 + 0.4(20) + \frac{0.4(0.4-1)}{2!}(-5) + \frac{0.4(0.4-1)(0.4-2)}{3!}(2) + \frac{0.4(0.4-1)(0.4-2)(0.4-3)(-3)}{4!}$$

$$f(x) = 46 + 0.4(20) + \frac{0.4(-0.6)}{2}(-5) + \frac{0.4(-0.6)(-1.6)}{6}(2) + \frac{0.4(-0.6)(-1.6)(2.6)}{8}$$

$= 46+8+0.6+0.128+0.1248$

$=54.8528$

To find f(1925):

Here $x_n = 1931, y_n = 101, \nabla y_n = 8, \nabla^2 y_n = -4, \nabla^3 y_n = -1, \nabla^4 y_n = -3,$

$$u = \frac{x - x_n}{h}$$

$$u = \frac{1925-1931}{10}$$

$= -0.6$

By NBIF is

$$Y_u = y_n + u\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n + \ldots\ldots + \frac{u(u+1)\ldots..(u+(n-1))}{n!}\nabla^n y_n$$

$$f(1925) = 101 + \frac{(-0.6)(-0.6+1)}{2!}(-4) + \frac{(-0.6)(-0.6+1)(-0.6+2)}{3!}(-1)$$

$$+ \frac{(-0.6)(-0.6+1)(-0.6+2)(-0.6+3)}{4!}(-3)$$

$$= 101 + (-0.6)(0.4)(-2) + (-0.6)(8) + \frac{(0.6)(0.4)(1.4)}{6} + \frac{(0.6)(0.4)(1.4)(2.4)}{8}$$

=101+0.48-4.8+0.056+0.1008

=96.8368

2. For the following are the no of deaths in 4 successive 10 year age group.Find the no of deaths at 45-50 and 50-55 age group

Age groups(x):	25-35	35-45	45-55	55-65
Deaths:	13229	18139	24225	31496

Sol:

Difference table

Age group(<x)	Deaths	Δ	Δ^2	Δ^3
35	13229			
		18139		
45	31368		6086	
		24225		1185
55	55593		7271	
		31496		
65	87089			

From table

$X_0 = 35, y_0 = 13229$, $\Delta y_0 = 18139$, $\Delta^2 y_0 = 6086$, $\Delta^3 y_0 = 1885$

$X = 50$, $h = 10$, $u = \frac{x - x0}{h} = \frac{50-35}{10} = 1.5$

By NFIF

$$Y_u = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0$$

$$= 13229 + 1.5(18139) + \frac{(1.5)(1.5-1)(6086)}{2!} + \frac{(1.5)(1.5-1)(1.5-2)}{3!}(1185)$$

=13229+27208.5+0.375(6086)-0.0625(1185)

=13229+27208.5+2282.25-74.0625

=426465.6875

=42646 (approx)

The no of deaths less than 55 years is 42646

The no of deaths b/w 45 and 50 = 42646-31368

=11278

The no of deaths between 50-55 =5593-42646

=12947

3. Tables gives the distance between nautical miles of the visible horizon for the given height in feet above the earth surface

Height(x):	100	150	200	250	300	350	400
Distance(Y):	10.63	13.03	15.04	16.81	18.42	19.9	21.27

Sol:Difference table

X	Y	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
100	1063						
		2.4					
150	13.03		-0.39				
		2.01		0.15			
200	15.04		-0.24		0.07		
		1.77		0.08		0.02	
250	16.81		-0.16		-0.05		0.02
		1.61		0.03		0.04	
300	18.42		-0.13		-0.01		
		1.48		0.02			
350	19.9		-0.11				
		1.37					
400	21.27						

From the above table ,

$y_0=15.04, \Delta y_0=1.77, \Delta^2 y_{-1} = -0.24, \Delta^3 y_{-1} = 0.08, \Delta^4 y_{-2} = 0.07, \Delta^5 y_{-2} = 0.02, h=50,$

$$u = \frac{x - x0}{h}$$

$$u = \frac{218 - 200}{50}$$

=0.36

By GFIF

$$Y_u = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_{-1} + \frac{u(u+1)(u-1)}{3!}\Delta^3 y_{-1} + \frac{u(u+1)(u-1)(u-2)}{4!}\Delta^4 y_{-2} + \ldots\ldots\ldots$$

$$= 15.04 + (0.36)(1.77) + \frac{(0.36)(0.36-1)(-0.29)}{2!} + \frac{(0.36)(0.36-1)(0.36+1)(0.08)}{3!} + \frac{(0.36)(0.36-1)(0.36+1)(0.36-2)}{4!}(0.07) + \frac{(0.36)(0.36-1)(0.36+1)(0.36-2)(0.36+2)}{5!}(0.02)$$

$$= 15.04 + 0.06372 + \frac{(-0.2304)}{2}(-0.29) + \frac{(-0.3133)}{6}(0.08) + \frac{(0.36)(-0.64)(1.36)(1.64}{24}(0.07) + \frac{(0.36)(-0.64)(1.36)(-1.64)(2.36)}{120}(0.02)$$

$=15.04+0.06372-0.1152(-0.29)-(0.0522)(0.08)+0.0214(0.07)+0.0101(0.02)$
$=15.04+0.637+0.0334-0.004176+0.001498+0.000202$
$=15.6781$

4. Interpolate by means of Gauss' back ward interpolation formula the sales of concern of the year 1976

Year(x):	1940	1950	1960	1970	1980	1990
Sales (in Lakhs):	17	20	27	32	36	38

Sol: let us take $x_0=1970$

Difference table

x	Y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1940	17					
		3				
1950	20		4			
		7		-6		
1960	27		-2		7	
		5		1		-9
1970	32		-1		-2	
		4		-1		
1980	36		-2			
		2				
1990	38					

$y_0=32$, $\Delta y_{-1}=5$, $\Delta^2 y_{-1}=-1$, $\Delta^3 y_{-2}=1$, $\Delta^4 y_{-2}=-2$, $\Delta^5 y_{-3}=-9$

Let $x=1976, x_0=1970$

$h=10$ then

$$u=\frac{x-x_0}{h}$$
$$=\frac{1976-1970)}{10}$$
$$=\frac{6}{10}$$
$$=0.6$$

From the Gauss's Backward difference formula we have

$$Y_u=y_0+u\Delta y_{-1}+\frac{u(u+1)\Delta^2 y_{-1}}{2!}+\frac{u(u+1)(u-1)\,\Delta^3 y_{-1}}{3!}+\frac{u(u+1)u(u-1)(u+2)\,\Delta^4 y_{-2}}{4!}+\ldots\ldots\ldots$$

$$=32+(0.6)(5)+\frac{(0.6)(06+1)(-1)}{2}+\frac{(0.6+1)(0.6-1)(0.6)(1)}{6}$$
$$+\frac{(0.6+1)(0.6-1)(0.6)(0.6+2)(-2)}{24}+\frac{(0.6+1)(0.6-1)(0.6)(0.6+2)(0.6-2)+}{120}($$
$$=32+3-\frac{0.96}{2}-\frac{0.384}{6}+\frac{1.9968}{24}-\frac{12.5798}{120}$$
$$=32+3-0.48-0.064+0.0832-0.1048$$
$$=34.4344$$

5.For the following are the percentage of poverty for successive 5 yaers

Year(x):	1985	1990	1995	2000	2005
Poverty(Percentage):	60	56	51	45	40

Find the percentage of poverty during the year 1996.

Sol:

X	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1985	60				
		-8			
1990	52		-1		
		-9		12	
1995	43		11		-30
		2		-18	
2000	45		--7		
		-5			
2005	40				

From table

$y_0=43,\ \Delta y_{-1}=-9,\ \Delta^2 y_{-1}=11,\ \Delta^3 y_{-2}=12,\ \Delta^4 y_{-2}=30$

$$u=\frac{x-x0}{h}$$

$$=\frac{1996-1995}{5}$$

$$=0.2$$

From theStrilings formula we have

$$_u= y_0+\frac{u[\Delta y_0+\Delta y_{-1}]}{2}+\frac{u^2}{2!}\Delta^2 y_{-1}+\frac{(u^2-1)u}{3!}\frac{[\Delta^3 y_{-1}+\Delta^3 y_{-2}]}{2}+\frac{(u^2-1)}{4!}u^2\Delta^4 y_{-2+\ldots\ldots}$$

$$=43+(0.2)\left(\frac{2(-9)}{2}\right)+\frac{0.2^2}{2!}(11)+\frac{(0.2^2-1)}{3!}0.2+\left[\frac{-18+12}{2}\right]\frac{(-0.96)0.04+}{24}(-30)$$

$=43+0.2(-3.5)+0.02(11)-(0.16)(0.2)(-3)-0.0016(-30)$

$=43+(-0.7)+0.22+0.096+0.048$

$=43-0.7+0.22+0.096+0.048$

$=42.664$

6.Following data gives the melting point of an alloy of lead and zinc

percentage of an alloy	50	60	70	80
Temparature ©	205	225	248	274

Find the melting of the alloy containing 54% of lead, using approx interpolation formula

Sol:

Let $x = 54, x_0 = 50$

$h = 10$ then

$$u = (x - x0)/h$$
$$= \frac{54-50}{10}$$
$$= \frac{4}{10}$$
$$= 0.4$$

Difference table

X	y	Δy	$\Delta^2 y$	$\Delta^3 y$
50	205			
		20		
60	225		3	
		-23		0
70	248		3	
		26		
80	274			

From table

$\Delta y_0=20$, $\Delta^2 y_0= 3$, $\Delta^3 y_0= 0$

By NFIF

$$Y_u= y_0+u\Delta y_0+\frac{u(u-1)}{2!}\Delta^2 y_0 +\frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 +\frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0$$

$$= 205 + 0.4(20) + \frac{(0.4)(0.4-1)}{2}(3) + \frac{(0.4)(0.4-1)(0.4-2)}{6}(0)$$

$$= 205 + 8 + \frac{(0.4)(-0.6)}{2}(3) + \frac{(0.4)(-0.6)(-1.6)}{6}(0)$$

$$= 205 + 8 - \frac{0.72}{2}$$

$$= 205 + 8 - 0.36$$

$$= 212.64$$

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